

FINANCIAL LEVERAGE IN THE CASE OF AN INVESTMENT PROJECT WITH STOCHASTIC CASH FLOW**Andrejs Jaunzems***Ventspils University College**E-mail: jaunzems@venta.lv*

The management science examines the investment and financing as business actions what generate intertemporal input-output cash flows. The decisions about investment and financing are important long-run decisions based on the prediction how money borrowing and lending and created cash flow will influence the business processes. In order to make rational decisions investor applies wide spectrum of the quantitative analysis methods. Information and knowledge transform the preferences of investor and is an important tool for uncertainty and risk reduction. However, the analysis of literature (see, for example, [1], [2], [3], [4], [5], [6]) indicates that the criterion for accepting long-term investments is the most controversial issue in financial management. *James C. Van Horne, John M. Vachowiz Jr.* state "Criterion for accepting long-term investments is probably the most controversial and difficult issue in financial management." In this paper different attributes of the intertemporal cash flows generated by investment and financing programs which influence the investor's utility are investigated. A case of stochastic investment cash flows is considered. The main result is formulated as rule of credit contract selection: at a given amount of borrowed sum and given interest rate the contract with the largest interest enlarges the expected net present value of the created stochastic cash flow and reduces the probability that the net present value will become negative. The original concept of the credit contract scissors is introduced. The presentation of the material is illustrated with an example calculated using Microsoft Excel. The results of the present paper appear to be innovative, not discussed in literature available to the author of the present paper. The theoretical questions examined in this paper require further investigation.

1. The stochastic cash flow's net present value and internal rate of return as random variables.

Let us consider the stochastic cash flow $X = (X_1, X_2, \dots, X_n)$, where X_1, X_2, \dots, X_n are the cash inflows or outflows corresponding to the moments t_1, t_2, \dots, t_n , from investor's mr. Johnson's point of view. The volume of money is measured, for example, in dollars; time is measured in years. The probability distribution of the n-dimensional stochastic vector X is given.

The initial investment is determined and equals x_0 . According to interpretation that is typical for microanalysis we assume, that in the present moment the investor has an opportunity to buy this stochastic intertemporal bundle of cash X for price x_0 .

Let us introduce notation $\mathbf{X} = (-x_0, X_1, X_2, \dots, X_n) = (-x_0, X)$.

For any particular investor the rate of return on capital (opportunity cost of capital, market capitalization rate) traditionally is applied as an indicator of time value of money.

However, in order to compare two investment projects investor takes into account broad context and different considerations.

In the case of determined cash flows, the theory of financial management offers the investor functions of interest rate that are useful for understanding the attributes of cash flows; these functions include net present value, benefit-cost ratio or profitability index, duration and volatility. In case of stochastic cash flow probability distributions of quantitative indicators are constructed. By applying stochastic models, uncertainty becomes measurable and is defined as a risk. Traditionally standard deviation is used as a risk indicator.

Let us associate discount factor v to the given annual interest rate i . The discount factor can be defined in different ways depending on the interest capitalization scheme chosen by investor.

We will use below the discount vector $T = (v^{t_1}, v^{t_2}, \dots, v^{t_n})$. The notation of the discount vector with letter T stresses that discounting is the most popular method to make a quantitative estimate of the influence of time factor.

Let us keep in mind that with regard to the scheme of interest capitalization the discount vector T unanimously corresponds to the interest rate i .

Remark. In the theoretical part, we will use only vector T . Exact definition of the discount factor v with respect to interest rate i in this part does not matter. In the example provided below quantitative results would be different, if instead of the discount factor $v = \frac{1}{1+i}$

another discount factor, for example, $v = (1 + \frac{i}{12})^{-1}$ or $v = e^{-i}$, would be used.

Let us denote the net present value of the stochastic cash flow $X = (X_1, X_2, \dots, X_n)$ as $NPV(X; i)$ and let us use the vectorial form: $NPV(X; i) := T \cdot X$.

Because of linearity of expected value operation expected net present value can be calculated by formula:

$$E[NPV(X; i)] = NPV[E(X); i] = T \cdot E(X), \text{ kur } E(X) := (E(X_1), E(X_2), \dots, E(X_n)).$$

According to the definition dispersion or variance of the net present value is:

$$D[NPV(X; i)] = T \text{ cov}(X) T^T.$$

Here $\text{cov}(X) := (\text{cov}(X_i, X_j))$ is the $(n \times n)$ -matrix of covariations between components of stochastic cash flow X ; T^T is the transposed discount vector T .

$$\text{Thus, } \text{cov}(X) \in \mathbf{R}^{n, n}, T \in \mathbf{R}^{1, n}, T^T \in \mathbf{R}^{n, 1}.$$

The standard deviation of the net present value is denoted by $SD[NPV(X; i)]$.

The functions $E [NPV(X; i)]$, $SD [NPV(X; i)]$ of interest rate i will be used to characterize stochastic cash flow $X = (X_1, X_2, \dots, X_n)$.

The graphs of these two functions depicted in coordinate plain (abscissa: interest rate; ordinate: amount of money) allow us to obtain a clear image about the variation of the expected net present value and risk with respect to interest rate's variations.

In the well-known Harry Markowitz diversification model the security with stochastic return R is depicted as point $(SD(R), E(R))$. In analogy with Harry Markowitz diversification model in order to visualize the attributes of stochastic cash flow X we offer graphical picture of the trajectory $\{(SD[NPV(X; i)], E[NPV(X; i)]) \mid i \in [0, 1]\}$.

In case of determined cash flow investor is interested to know internal rate of return IRR, which is a meaningful financial indicator. Internal rate of return I^* of the stochastic cash flow X is random variable, defined as a solution to equation $NPV(X; I^*) = x_0$.

$$\text{Obviously } NPV(E(X); E(I^*)) = x_0, \text{ or } E(I^*) = IRR(-x_0, E(X)).$$

The probability distribution function of random variable I^* is

$$Prob \{I^* \leq i\} = Prob \{i \mid NPV(X; i) \leq x_0\}, i \geq 0.$$

2. The influence of the financial leverage on the indicators of the consolidated stochastic cash flow and on the decision of investor.

We consider above the stochastic cash flow $X = (X_1, X_2, \dots, X_n)$ which investor mr. Johnson can afford to buy for x_0 dollars.

Let us assume now that investor is able to borrow amount of money k_0 , which must be returned in the form of determined cash flow (debt) $D = (d_1, d_2, \dots, d_n)$ defined in credit contract. Here nonnegative numbers d_1, d_2, \dots, d_n are amounts of money paid back in time moments t_1, t_2, \dots, t_n .

Remark. We make the unification of sequence of terms if necessary.

In order to have simpler expressions we will speak about cash flow D as about the payback contract of the loan k_0 .

$$\text{The notation } K := (k_0, -d_1, -d_2, \dots, -d_n) = (k_0, -D) \text{ also will be used.}$$

Let us denote the interest rate of credit by r . Thus $k_0 = NPV(D; r)$, or $r = IRR(-k_0, D)$.

Let us assume that investor decides to invest money and to borrow money. In this case investor gets stochastic consolidated intertemporal money bundle

$$X + K = (-x_0 + k_0, x_1 - d_1, x_2 - d_2, \dots, x_n - d_n) = (-x_0 + k_0, X - D).$$

In this case, we will say that investor consumes financial leverage, *финансовый рычаг*.

We will investigate the influence of the credit interest rate r and design of credit contract D on the investors' decision.

Let us assume, that investor Mr. Johnson owns x_0 dollars. Investor has an opportunity to buy stochastic intertemporal money bundle X for his present x_0 dollars. If an opportunity for the investor arises to buy from bank present k_0 dollars for his intertemporal determined cash flow D , than as result of both exchanges, investor Mr. Johnson gets an opportunity to buy consolidated intertemporal stochastic cash bundle $X - D$ for his present $x_0 - k_0$ dollars.

What decision will investor make?

Investor compares utilities of the alternative goods:

$u(x_0)$, $u(X)$, $u(x_0 - k_0)$, $u(X - D)$, $u(\mathbf{X})$, $u(\mathbf{X} + K)$.

Obviously, according to consumers' preferences property "more is better" and the following inequalities fulfil:

$u(x_0) > u(x_0 - k_0)$, $u(X) > u(X - D)$.

(1) If $u(x_0) > u(X)$ and $u(x_0 - k_0) > u(X - D)$, than investor keeps his x_0 dollars and does not take part in the project.

(2) If $u(x_0) < u(X)$ and $u(x_0 - k_0) > u(k_0, X - D)$, than investor invests his x_0 dollars in the project in order to get X , but does not borrow.

(3) If $u(x_0) > u(X)$ and $u(x_0 - k_0) < u(X - D)$, than investor borrows k_0 dollars and invests his own $x_0 - k_0$ dollars in the project in order to get $X - D$.

(4) If $u(x_0) < u(X)$, $u(x_0 - k_0) < u(X - D)$ and $u(-x_0; X) > u(-x_0 + k_0; X - D)$, than investor invests his x_0 dollars in the project in order to get X , but does not borrow.

(5) If $u(x_0) < u(X)$, $u(x_0 - k_0) < u(X - D)$ and $u(-x_0; X) < u(-x_0 + k_0; X - D)$, than investor borrows k_0 dollars and invests his own $x_0 - k_0$ dollars in the project in order to get intertemporal stochastic money bundle $X - D$.

Let us introduce discount vector $T := (v^{t_1}, v^{t_2}, \dots, v^{t_n})$.

The net present value of the consolidated stochastic cash flow $X - D = (X_1 - d_1, X_2 - d_2, \dots, X_n - d_n)$ is a random variable $NPV(X - D; i) = T \cdot X - T \cdot D$ with expected value $E[NPV(X - D; i)] = T \cdot E(X) - T \cdot D$ and dispersion $D[NPV(X - D; i)] = T \cdot cov(X) \cdot T^T$.

The standard deviation of the net present value is denoted by $SD[NPV(X - D; i)]$.

Internal rate of return I^{**} of the consolidated stochastic cash flow $\mathbf{X} + K = (-x_0 + k_0, X - D)$ is a random variable, defined as a solution to equation $NPV(X - D; I^{**}) = x_0 - k_0$.

Obviously $NPV(E(X - D); E(I^{**})) = x_0 - k_0$, or $E(I^{**}) = IRR(-x_0 + k_0, E(X) - D)$.

The probability distribution function of the random variable I^{**} is a function, that for each interest rate $i \geq 0$ allows to calculate probability $\{ Prob\{ NPV(X - D; i) \leq x_0 - k_0 \}$.

The result of research is formulated in the form of a theorem.

Theorem. Let us consider financial leverages in a form $K := (k_0, -d_1, -d_2, \dots, -d_n)$, where $k_0 = NPV(D; r)$, and corresponding consolidated stochastic cash flows are in a form

$\mathbf{X} + K = (-x_0 + k_0, x_1 - d_1, x_2 - d_2, \dots, x_n - d_n)$.

The expected return increases more, if \mathbf{X} is consolidated with a credit contract K requiring interest. Financial leverage leads to decreased investment risk: the probability of the situation, when consolidated stochastic cash flow's rate of return is less than fixed interest rate $i > r$, decreases. Besides this probability decreases more if \mathbf{X} is consolidated with credit contract K requiring larger interest.

We will support this theorem with the help of geometrical visualization, providing an example.

3. Example.

3.1. The quantitative indicators of stochastic cash flow $\mathbf{X} = (-x_0, X_1, X_2, X_3)$ associated with investment project.

By making initial investment $x_0 = 100$ dollars investor mr. Johnson is able to purchase stochastic cash flow $X = (X_1, X_2, X_3)$.

Let us assume, that three-dimensional stochastic vector X is normally distributed, $E(X) = (40, 60, 50)$. The correlation matrix and covariation matrix of stochastic vector X are given in table 1.

Table 1: The correlation matrix and covariation matrix of stochastic vector X .

	1,0	0,6	0,3		16,0	14,4	10,8
$cor(X_1, X_2, X_3) =$	0,6	1,0	0,8	$cov(X_1, X_2, X_3) =$	14,4	36,0	43,2
	0,3	0,8	1,0		10,8	43,2	81,0

Let us use the discount vector $T: = (v, v^2, v^3)$, where $v = (1 + i)^{-1}$.

For given stochastic cash flow $X = (X_1, X_2, X_3)$ we have:

the net present value $NPV(X; i) = T \cdot X$,

the variance $D[NPV(X; i)] = T cov(X) T^T = (v, v^2, v^3) cov(X_1, X_2, X_3) (v, v^2, v^3)^T$.

In figure 1, the graph of expected net present value $E[NPV(X; i)]$ is depicted. The one-, two- and three standard deviation lines around expected net present value are also depicted. The picture gives us a clear image about probability distribution of $NPV(X; i)$ and about probability that value of $NPV(X; i)$ can be less than 100.

For example, $E[NPV(X; 0,10)] = 123,52$; $SD[NPV(X; 0,10)] = 13,17$.

Corresponding one standard deviation interval is $[110, 34; 136, 69]$, two standard deviation interval is $[97, 17; 149,87]$, three standard deviation interval is $[83,99; 163,04]$.

It means that at a given interest rate $i = 10\%$ stochastic net present value $NPV(X; 0,10)$ belongs to the mentioned intervals with following probabilities:

$$Prob\{ 110,34 < NPV(X; 0,10) < 136,69 \} = 0,683;$$

$$Prob\{ 97,17 < NPV(X; 0,10) < 149,87 \} = 0,955;$$

$$Prob\{ 83,99 < NPV(X; 0,10) < 163,04 \} = 0,997.$$

In figure 2, the graphs of stochastic cash flow's $X = (-100, X)$ expected net present value $E(NPV(X; i))$ and standard deviation $SD(NPV(X; i))$ as functions of interest rate i are depicted.

In table 2, the values of expected net present value and standard deviation of stochastic cash flow $X = (-100, X)$ with respect to interest rate i are showed.

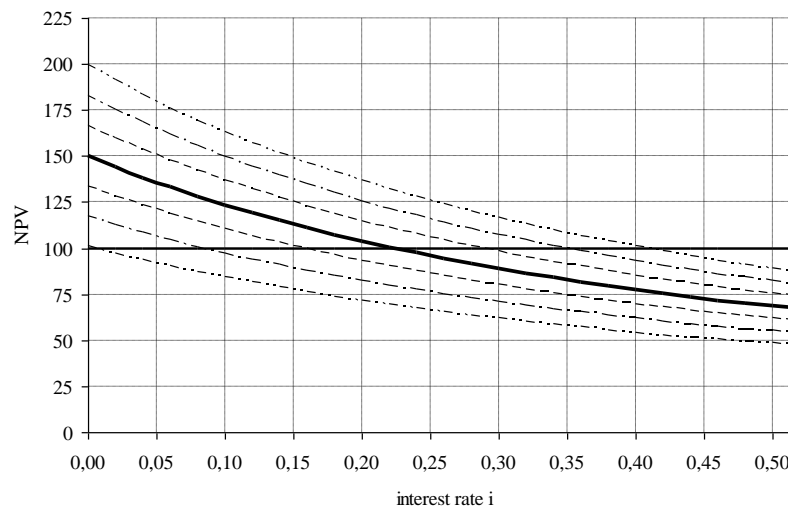


Figure 1: Graph of expected net present value $E[NPV(X; i)]$. One-, two- and three standard deviation stripes around expected net present value graph.

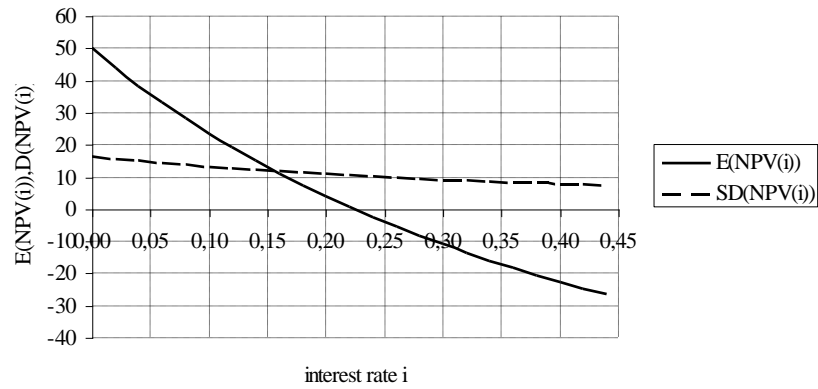


Figure 2: Graphs of stochastic cash flow's $X = (-100, X)$ expected net present value $E(NPV(X; i))$ and standard deviation $SD(NPV(X; i))$ as functions of interest rate i .

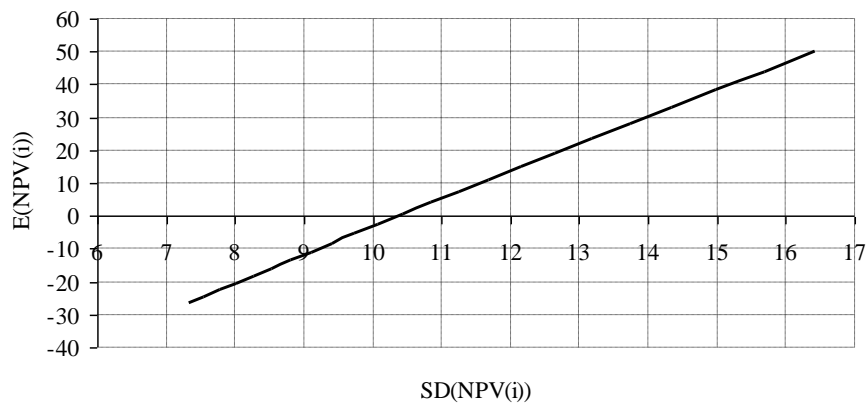


Figure 3: Graph of the trajectory $\{ (SD(NPV(X; i)), E(NPV(X; i)) | i \in [0, 0,40] \}$.

Table 2: The points of trajectory $\{ (SD(NPV(X; i)), E(NPV(X; i)) | i \in [0, 0,40] \}$.

i	0,00	0,02	0,04	0,06	0,08	0,10	0,12	0,14	0,16	0,18	0,20	0,22	0,24	0,26	0,28	0,30
$SD(NPV(X; i))$	16,43	15,68	14,99	14,34	13,74	13,17	12,65	12,15	11,68	11,25	10,84	10,45	10,08	9,74	9,41	9,10
$E(NPV(X; i))$	50,00	44,00	38,38	33,12	28,17	23,52	19,13	15,00	11,11	7,42	3,94	0,63	-2,50	-5,47	-8,29	-10,97

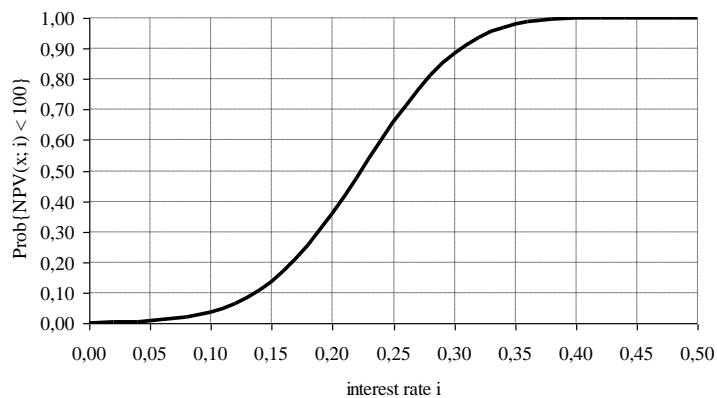


Figure 4: Graph of probability distribution function $Prob \{I^* \leq i\} = Prob\{ i | NPV(X; i) \leq x_0 \}$, $i \geq 0$, of cash flows' $X = (-100, X)$ internal rate of return I^* .

3.2. The quantitative attributors of the consolidated stochastic cash flow $\mathbf{X} + \mathbf{K} = (-x_0 + k_0, X_1 - d_1, X_2 - d_2, X_3 - d_3)$.

Let us continue considering our example. Let us assume that investor mr. Johnson decides to support his own cash flow \mathbf{X} with loan $k_0 = 50$ under interest rate $r = 15\%$. Credit contract lasts three years. Mr. Johnson will be considered as investor, and lender will be recognized as creditor.

We will consider solely three – year loans under 15% interest rate. We will investigate variations of quantitative attributors of consolidated stochastic cash flow with respect of the credit contracts' form.

In order to illustrate the problem let us consider two kinds of credit extinguishing contracts.

The first contract is denoted by D1. According this contract both the principal and interest must be paid in the end of the third year with joint payment 76,04 dollars. According to contract D1 50 dollars of debt are extinguished in three years proposing interest rate 15% with the largest summary interest. Interest is 26,04 dollars.

The second contract is denoted by D2. This is the 50 dollars debt amortization (annuity) contract, when debt is extinguished with three equal 21,90 dollars payments. Interest is just 15, 70 dollars.

Let us investigate the consolidated cash flows $\mathbf{X} - \mathbf{D1}$, $\mathbf{X} - \mathbf{D2}$.

Consolidation of the investment project's cash flow \mathbf{X} with certain credit cash flow we interpret as investment project support with credit.

Which contract does support the investor's Johnson's investment project better?

In table 3 the cash flows and their internal rates of return are shown. Let us observe that expected internal rate of return of the consolidated cash flow $(-50, \mathbf{X} - \mathbf{D1})$, $(-50, \mathbf{X} - \mathbf{D2})$ substantially exceeds the cash flow's $(-100, \mathbf{X})$ expected internal rate of return. Rate of expected return of the cash flow $(-50, \mathbf{X} - \mathbf{D1})$ is especially high.

Table 3: The cash flows \mathbf{X} , $\mathbf{K1}$, $\mathbf{K2}$, $\mathbf{X} + \mathbf{K1}$, $\mathbf{X} + \mathbf{K2}$ and their expected internal rates of return

Cash flow	t = 0	t = 1	t = 2	t = 3	E(IRR)
$\mathbf{X} := (-x_0, E(\mathbf{X}))$	-100	40	60	50	0,22
$\mathbf{K1} := (k_0, -\mathbf{D1})$	50	0	0	-76,04	0,15
$\mathbf{K2} := (k_0, -\mathbf{D2})$	50	-21,90	-21,90	-21,90	0,15
$\mathbf{X} + \mathbf{K1}$	-50	40	60	-26,04	0,39
$\mathbf{X} + \mathbf{K2}$	-50	18,10	38,10	28,10	0,29

Theorem 1. The expected internal rate of return increases more in case of its consolidation with credit contract proposing largest interest.

We will consider the geometrical proof of this theorem with help of picture Nr. 5.

The graphs of the cash flows' expected net present values depicted in picture Nr. 5 geometrically confirm the fact that expected internal rate of return increases more in case of consolidation with credit contract proposing largest interest. Analytically, this conclusion follows from linearity of the net present value as cash flow function. Because of linearity the graph of expected net present value of consolidated cash flow $\mathbf{X} + \mathbf{K1}$ can be calculated as a sum of the graphs of net present value of cash flow $E(\mathbf{X})$ and cash flow $\mathbf{K1}$:

$$NPV(E(\mathbf{X}) + \mathbf{K1}; i) = NPV(E(\mathbf{X}); i) + NPV(\mathbf{K1}; i).$$

Analogically the graph of expected net present value of cash flow $\mathbf{X} + \mathbf{K2}$ can be calculated as sum of the graphs of net present value of cash flow $E(\mathbf{X})$ and cash flow $\mathbf{K2}$:

$$NPV(E(\mathbf{X}) + \mathbf{K2}; i) = NPV(E(\mathbf{X}); i) + NPV(\mathbf{K2}; i).$$

The 15% interest rate credit contracts' K1, K2 net present value graphs both cross the abscissa axis at point where $i = 0,15$. The interest of credit geometrically can be observed as an ordinate (by modulus) of curve where $i = 0$.

Because $NPV(K1; i) > NPV(K2; i) \forall i > 0,15$ from equality $NPV(E(X); i) + NPV(K2; i) = 0$ follows inequality $NPV(E(X); i) + NPV(K1; i) > 0$.

Thus $IRR [E(X) + K1] > IRR [E(X) + K2]$.

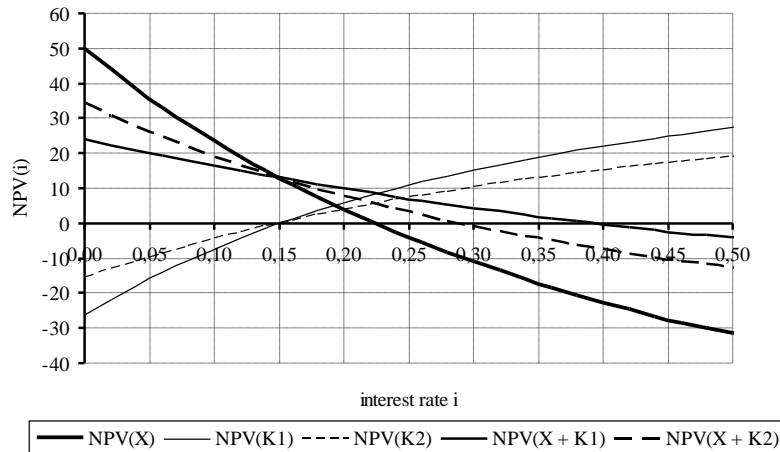


Figure 5: The graphs of expected net present values of the cash flows used in example.

Theorem 2. The financial leverage reduces investment risk in the following way: the probability that internal rate of return of consolidated cash flow is less than certain interest rate $i > r$, decreases. Furthermore, this reduction is larger in case of consolidation with credit contract proposing larger interest.

Let us consider the theorem's foundation.

The first, let us remark, that for any $i \geq 0$ the standard deviation of the consolidate cash flow's net present value is the same as standard deviation of the initial cash flow X. Indeed, by definition of dispersion the following equality holds: $D[NPV(X - D; i)] = T cov(X) T^T = D[NPV(X; i)]$.

Because of that the one-, two- and three- standard deviation strips around graph of $NPV(E(X) + K1; i)$ are exactly as wide as they are around the graph of $NPV(E(X); i)$. The graph of $NPV(E(X) + K1; i)$ for any $i > r$ is located above the graph of $NPV(E(X); i)$. Therefore these stripes cross axis of interest rate starting from larger interest rate values.

In the figure 6 the graphs of the functions $NPV(E(X); i)$, $NPV(E(X) + K1; i)$ are depicted. One-, two- and three- standard deviation stripes around graph of $NPV(E(X) + K1; i)$ are shown.

The variations of the rate of return risk under influence of financial leverage are especially well demonstrated in picture 7, where the graphs of operative characteristics of return for consolidated cash flows $(-50, X - D1)$, $(-50, X - D2)$, $(-100, X)$ are depicted.

For example, the following points of graphs $(0, 10; 0, 11)$, $(0, 10; 0, 07)$, $(0, 10; 0, 04)$ mean:

the probability, that internal rate of return of the consolidated cash flow $(-50, X - D1)$ is less than 10%, is 0,11;

the probability, that internal rate of return of the consolidated cash flow $(-50, X - D2)$ is less than 10%, is 0,07;

the probability, that internal rate of return of the cash flow $(-100, X)$ is less than 10%, is 0,04.

Whereas the following points of graphs $(0,20; 0,18)$, $(0,20; 0,23)$, $(0,20; 0,36)$ mean:

the probability, that internal rate of return of the consolidated cash flow $(-50, X - D1)$ is less than 20%, is 0,18;

the probability, that internal rate of return of the consolidated cash flow $(-50, X - D2)$ is less than 20%, is 0,23;

the probability, that internal rate of return of the cash flow $(-100, X)$ is less than 20%, is 0,36.

The interest rate $i = r = 0,15$ is interpreted as a special margin:

if the opportunity cost of capital i in the financial market is less than credit's interest rate r , than the borrowing of money reduces the gain to investor;

if the opportunity cost of capital i is larger than r , the borrowing of money benefits the investor.

The figure 7 demonstrates the special role of interest rate $i = r$:

in case $i \leq r$ the financial leverages $D1, D2$ lead to increased ordinates of probability distribution functions of the internal rates of return;

in case $i > r$ the financial leverages $D1, D2$ lead to decreased of ordinates of probability distribution functions of the internal rates of return.

In figure 8, the trajectories $\{(SD(NPV(\mathbf{X}; i)), E(NPV(\mathbf{X}; i)) \mid i \in [0, 0, 40]\}$,

$\{(SD(NPV(\mathbf{X} + K1; i)), E(NPV(\mathbf{X} + K1; i)) \mid i \in [0, 0,40]\}$ are depicted. The crosspoint of the trajectories corresponds to the interest rate $i = 0,15 = r$.

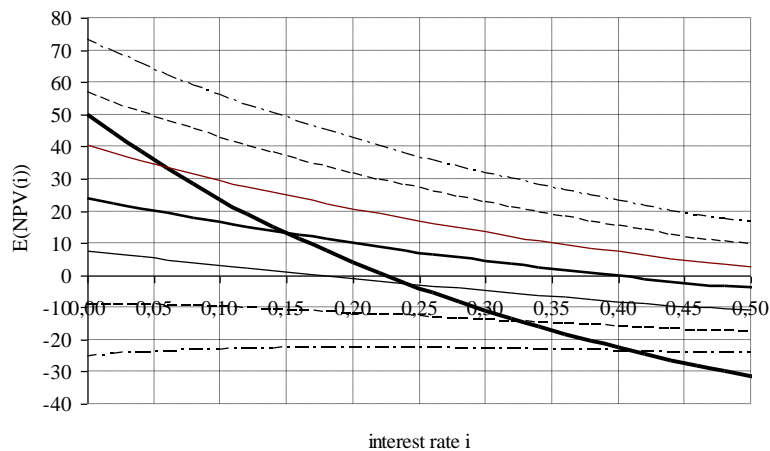


Figure 6: The graphs of the functions $NPV(E(X); i)$, $NPV(E(X) + K1; i)$. One-, two- and three standard deviation stripes around graph of $NPV(E(X) + K1; i)$ are shown.

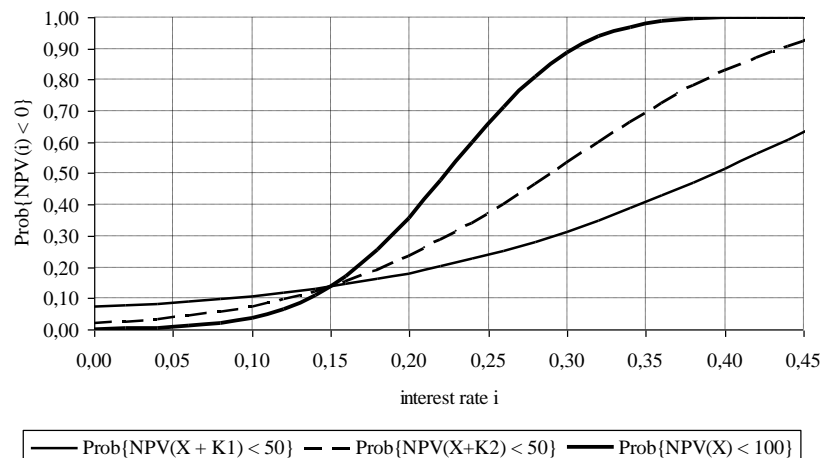


Figure 7: The graphs of probability distribution functions of the cash flows' $(-100, X)$, $(-50, X - D1)$, $(-50, X - D2)$ internal rate of return.

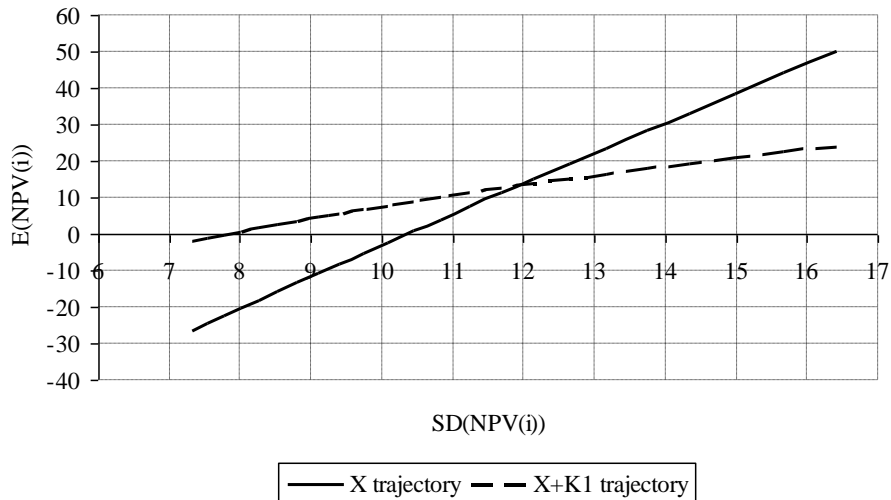


Figure 8: The graphs of trajectories of the stochastic cash flows X and $X + K1$.

If $i > 0,15 = r$, the risk reducing effect is present: for every risk SD the expected return of the cash flow $X + K1$ exceeds the expected return of cash flow X .

If $i < 0,15 = r$, than the risk of investor increases.

3.3. The concept of the credit contract scissors.

Let us consider 50 dollars loans $K1$ and $K2$ with a three year term and proposing different interest rates r .

Let us calculate expected returns of the consolidated cash flows $X + K1$, $X + K2$ with respect to credit interest rate r . The results of calculations are showed in the table 4 and geometrically are illustrated in the picture 9.

Table 4: The expected internal rate of return of the investment project, which is supported by credit cash flows $K1$, $K2$, with respect to credit interest rate r .

r	0,05	0,06	0,07	0,08	0,09	0,10	0,11	0,12	0,13	0,14	0,15	0,16	0,17	0,18	0,19	0,20	0,21	0,22	0,23	0,24	0,25
IRR(X)	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22	0,22
IRR(X + K1)	0,52	0,51	0,50	0,49	0,48	0,46	0,45	0,44	0,42	0,41	0,39	0,38	0,36	0,34	0,32	0,29	0,27	0,24	0,20	0,16	0,11
IRR(X + K2)	0,37	0,37	0,36	0,35	0,34	0,33	0,32	0,32	0,31	0,30	0,29	0,28	0,27	0,26	0,25	0,25	0,24	0,23	0,22	0,21	0,20

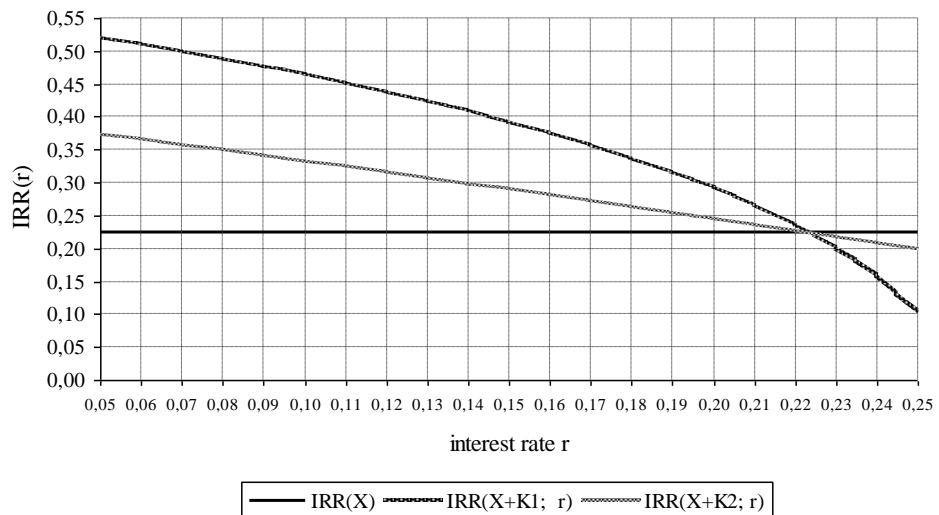


Figure 9: Credit contract scissors.

Conclusions. The geometry of pictures 5 and 9 allows us to make substantial economical conclusions.

(1) Let us assume that we consider the credit contract proposing fixed interest rate r .

As we saw before, investor is interested to have contract with largest interest. Large interest proposes extinguishing of debt during late payments. Duration of such credit cash flow is larger and therefore the volatility of contracts fundamental value is larger than duration and volatility of the credit contract with the same interest rate r but less interest. For lender that means larger risk. The borrower prefers to offer larger interest rate $r' > r$ by condition, that form of the contract corresponds to the wishes of investor – namely, the interest is large enough.

In the definite diapasons more effective in sense of return is leveraging the investment project with credit proposing larger interest rate and large amount of interest than credit proposing small interest rate and small amount of interest. The numerical explanation of this assertion is contained in table 4, but geometrical explanation – picture 9.

The situation arises what we call credit contract scissors. From table 4 and picture 9 it follows that, for example, both the 20% interest credit in form K1 and 15% interest credit in form K2 increase the return of investment project till 29%.

Thus, if some creditor agrees to give, for example, 18% credit with larger interest, winners will be both: creditor and investor because the return of investment project will increase till 34%. If creditor had reason not to be afraid from risk and he gave 15% credit in form K1, return of investment project would increase till 39%.

The problems of investment risk analysis often lead us to the fundamental issues of the effective organization of the national economy. The problems concerning relations between creditors and producers become more and more topical.

(2) Let us consider now the investment project X as owned property and net present value $NPV(X; i)$, calculated at the opportunity cost of capital i , as a fundamental value of asset.

Let us assume for the purpose of certainty that $i = 18\%$.

Then expected fundamental value of asset is $E[NPV(X; 0,18)] = 7,42$.

At the same time, $E[NPV(X + K1; 0,18)] = 11,14$; $E[NPV(X + K2; 0,18)] = 9,81$.

Thus, if opportunity cost of capital is 18%, than expected fundamental value of an asset X is 7,42. If potential buyer is able to support property X with financial leverage K1, the expected fundamental value increases till 11,42. If the asset X is supported with financial leverage K2, the expected fundamental value is 9,81.

During analysis of asset's fundamental value, the area between graphs of functions $E[NPV(X; i)]$ and $E[NPV(X+K1; i)]$, $E[NPV(X+K2; i)]$ in picture 5 is of interest to us. Let us assume that the opportunity cost of capital is 18% and asset X is bought for 7, 42, but buyer knows that he will be able to borrow money in form K1. Than the buyer will earn 25%.

It means that while estimating the fundamental value of an asset using discounting method of inflowing cash flow, it is not enough to operate solely with opportunity cost of capital, but it is necessary to take in account possibilities to support the asset with credit. And not only the cost of capital, but also the form of credit contract plays a huge role.

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