IMMUNIZED QUASI-ARBITRAGE

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Abstract

Arbitrage refers to the possibility of making money with no outlay of capital or possibility of loss. In this paper the concept of quasi-arbitrage is introduced: "A small risk, small net investment strategy that still generates significant profits". From existence of quasi-arbitrage the important consequences result, among them the "one price principle" failure necessary follows. The method of integral immunization what differs from well known Redington immunization is offered, and the numerical illustration of immunized quasi-arbitrage in the stylized financing an investing situation associated with case investigated by Lutz Krushwitz is presented. The concept of integral immunization of quasi-arbitrage appear to be innovative, not discussed in literature available to the author of the present paper. The theoretical questions examined in this paper require further investigation.

Keywords: arbitrage, quasi-arbitrage, fundamental value, volatility, duration, Redington immunization, integral immunization

1. INTRODUCTION

In the significant financial management textbooks the concept of arbitrage is explained differently and incompletely. For instance, Zvi Bodie, Robert C. Merton in the book [1; page 556] give the following explaining[7]: "The arbitrage is the bying and selling the similar and equal worth financial assets in different markets in order to get garanteed profit do to price difference." Zvi Bodie, Alex Cane, Alan J. Marcus, Stylianos Perrakis, Peter J. Ryan [3] define arbitrage in a categorical form as possibility to earn profit do to zero investment and zero risk[8]: "A zero-risk, zero-net investment strategy that still generates profits." James C. Van Horne, John M. Vachowiz Jr. write[9] [4; 928. p.]: "The arbitrage is to finding of two assets, which in principle does no differ between each other in order to buy the cheapest and sell the more expensive." In fundamental financial management textbook of Eugene F. Brigham, Louis C. Gapenski [6] the concept of arbitrage is not included at all. It is possible to continue the considering of different definitions of arbitrage. It seems to the author that each person who deals professionaly with economic theory feels some discontent about mystery and insufficient theoretical justify of the concept of arbitrage.

The assumption about arbitrage impossibility in the rational agent’s behaviour theory leads to the so called "one price principle". "Arbitrage refers to the possibility of making money with no outlay of capital or possibility of loss. Modern markets provide ready access to trading information and trades may be quickly transacted. Consequently, arbitrage oportunities should be short-lived. If you assume that there are no arbitrage opportunities, then two investments that have exactly the same cashflows must have the same prices. This is the "law of one price" and is the fundamental principle of "no-arbitrage" pricing."[10] In economic theory the "one price principle" is often used as most important assumption for significant models construction.

[7] Zvi Bodie, Robert C. Merton. Finance. − Prentice Hall, Pearson Education Company, 2000.
Боди, Зви; Мертон Роберт. Финансы.: Пер. с англ. − Издательский дом "Вилямс", 2004.
[8] Zvi Bodie, Alex Cane, Alan J. Marcus, Stylianos Perrakis, Peter J. Ryan. Investments. Third Canadian Edition. − McGraw-Hill, Ryerson, 1999.
[9] James C. Van Horne, John M. Vachowiz Jr. Fundamentals of Financial Management. Eleventh Edition. "Prentice Hall", 2001. Джеймс К. Ван Хорн, Джон М. Вахович мл. Основы финансового менеджмента. Одиннадцатое издание.– Москва, Издательский дом "Вильямс", 2003.
[10] Leslie Jane Federer Vaaler, James W. Daniel. Mathematical Interest Theory. Second edition. − The Mathematical Association of America. Pearsons Prentice Hall, 2007.

Historically, the conviction about impossibility of arbitrage arrives from the Arrow-Debreu contingent consumption perfect competition market attributes [7], [8], one of them is assumption about symmetric information in the agents interactions[11]. However in reality asymmetric information in markets has existed, exist in nowadays and will exist in the future. In the securities market ones call the agent as arbiter if this agent wins systematically because of being better informed about market to compare with another investors, to pass over the silence inside information legality problem. The global confidence crisis requires understanding the consequences of asymmetry of information available to market agents.

By opinion of author in commercial calculus the pure arbitrage has to be accepted as impossible. Indeed, the business which allows to generate profits without investment and without risk seems to us absolutely non-realistic in real markets, for instance, because of transactions cost. In the same time in real markets pretty often we can observe the cases of ultra profitable investments. The analysis of real markets gives us the conviction about validity of concept of quasi-arbitrage. We introduce the concept of quasi-arbitrage similarly how Zvi Bodie and Alex Cane define arbitrage: "A small risk, small net investment strategy that still generates significant profits." No doubt about the quasi-arbitrage existence in real markets, for instance, because of asymmetric information of agents. From existence of quasi-arbitrage a lot of important consequences result, among them the "one price principle" failure necessary follows.

Below the concept of integral immunization is considered, the numerical illustration of quasi-arbitrage with help of stylized financing an investing situation associated with case investigated by Lutz Krushwitz[12] using commercial calculus approach is presented. After that the method of quasi-arbitrage integral immunization is demonstrated.

2. THE CONCEPT OF IMMUNIZATION AND ANTIMMUNIZATION

Classical priciple of capital theory is associated with name of Irving Fisher[13]: "Value today always equals future cash flow discounted at the opportunity cost of capital."

The famous formula of Irving Fisher needs to be made more precise. Zvi Bodie, Robert C. Merton use the term "asset's fundamental value"[14]. It must be stressed that market value of an asset and its fundamental value as usualy differs and exactly in such case if investor is able to recognize undervalued and overestimated assets could arise possibilities of quasi-arbitrage.

In order to introduce the concept of immunization let us consider specific problem of financing and investing.

Let us assume that at the initial moment the market capitalization rate is δ_0.

Suppose that financial manager has to make payments according liabilities flow L in order to repay the borrowed sum − present value of L, what we will denote as $V(L; \delta_0)$. The present value of cash flow we interpret as fundamental value of intertemporal money bundle associated with given cash flow. The manager operates with money borrowed from investors. Utilizing the sum $V(L; \delta_0)$ the manager constructs the assets flow A, with help of what he plans to make payments and repay debt according liabilities flow L.

By definition, the present value of cash flow A equals the present value of cash flow L:

$$V(A; \delta_0) = V(L; \delta_0) \text{ or } V(A-L; \delta_0) = 0.$$

Let us assume that interest rate δ changes and becomes equal to $\delta = \delta_0 + \Delta\delta$.

Present value $V(A-L; \delta_0 + \Delta\delta)$ of cash flow A−L can increases but can also decreases.

Definition. The cash flow A−L is called immunized by interest rate δ_0, if number $\varepsilon > 0$ exist such that for any $\delta \in\ ]\delta_0 - \varepsilon; \delta_0 + \varepsilon[$, $\delta \neq \delta_0$, the inequality $V(A; \delta) > V(L; \delta)$ fulfil.

[11] Milne, Frank. Finance theory and asset pricing. − Oxford University Press, 1995.
Eichberger, Jurgen and Harper, Ian. Financial Economics. − Oxford Universituy Press, 1997.
[12] Lutz Kruschwitz. Investitionsrechnung. 8., Neu bearbeitete Auflage. − R. Oldenbourg Wissenschaftsverlag, 2000. Лутц Крушвиц. Инвестиционные расчеты. – Питер, 2001.
[13] Irving Fisher. The Nature of Capital and Income. − New York, 1923.
[14] Zvi Bodie, Robert C. Merton. Finance. − Prentice Hall, Pearson Education Company, 2000.
Боди, Зви; Мертон Роберт. Финансы.: Пер. с англ. − Издательский дом "Вилямс", 2004.

The cash flow A−L is called antiimmunized by interest rate δ_0, if number $\varepsilon > 0$ exist such that for any $\delta \in]\delta_0 - \varepsilon; \delta_0 + \varepsilon[$, $\delta \neq \delta_0$, the inequality $V(A; \delta) < V(L; \delta)$ fulfil.

Comments:

1. We are speaking about immunization in a strong sense. Immunization in a nonstrong sense means that the nonstreng inequality $V(A; \delta) \geq V(L; \delta)$ holds for any $\delta \in]\delta_0 - \varepsilon; \delta_0 + \varepsilon[$.

2. The volatility[15] of the $V(A; \delta)$ equals volatitlity of the $V(L; \delta)$ in the point $\delta = \delta_0$.

3. Immunization is specific kind of investment in fixed income securities diversification what helps to avoid from the interest rate risk. If assets portfolio is immunized then fluctuations of interest rate leads to the fundamental value of assets flow exceeding the fundamental value of liabilities flow. In the same time, investigation of anti-imunization allows us to identify the weakest assets portfolio construction decisions.

Let us formulate the problem of immunization in details.

Suppose that payments-liabilities flow is $L = (l_1\ \ l_2\ \ ...\ \ l_n) \geq O$.

We will construct the assets flow $A = (a_1\ \ a_2\ \ ...\ \ a_n) \geq O$, with help of which we are going to make payments and repay debt, as portfolio synthesized from m nonnegative cash flows:

$B_1 = (b_{1\,1}\ \ b_{1\,2}\ \ ...\ b_{1\,n})$, $B_2 = (b_{2\,1}\ \ b_{2\,2}\ \ ...\ b_{2\,n})$, ... , $B_m = (b_{m\,1}\ \ b_{m\,2}\ \ ...\ b_{m\,n})$.

We can immagine the cash flows $B_1, B_2, ... , B_m$ as cash flows associated with some fixed income security, for instance, obligations (bills, notes, bonds).

As it was stressed before, the interest rate at initial moment is δ_0.

The fundamental values of cash flows $B_1, B_2, ... , B_m$ are $V(B_1; \delta_0), V(B_2; \delta_0), ... , V(B_m; \delta_0)$.

Using the borrowed sum $V(L; \delta_0)$ we buy x_1 units of cash flow B_1, x_2 units of cash flow B_2, ... , and x_m units of cash flow B_m.

As result we get portfolio cash flow $A = x_1 B_1 + x_2 B_2 + ... + x_m B_m$.

According assumption present value of assets flow equals present value of liabilities flow:

$V(A; \delta_0) = x_1 \cdot V(B_1; \delta_0) + x_2 \cdot V(B_2; \delta_0) + ... + x_m \cdot V(B_m; \delta_0) = V(L; \delta_0)$.

Comment: We assume in this model that cash flows are arbitrary divisible. It means that $x_1, x_2, ... , x_m$ are nonnegative real numbers. Our goal is to construct assets flow (to determine numbers $x_1, x_2, ... , x_m$) so as the cash flow A−L would be immunized against fluctuations of interest rate.

3. CRITICISM OF REDINGTON[16] IMMUNIZATION

As before let us assume that at the initial moment the market capitalization rate is δ_0.

Well known is so called Redington immunization or differential immunization [9], based on the net present value $V(A-L; \delta)$ local approximation with Taylors' formula:

$$V(A-L; \delta) = V(A-L; \delta_0) + V'(A-L; \delta_0) \cdot (\delta - \delta_0) + 0{,}5 \cdot V''(A-L; \delta_0) \cdot (\delta - \delta_0)^2 + o[(\delta - \delta_0)^2],$$

where the remainder $o[(\delta - \delta_0)^2]$ vanishes to the order higher then $(\delta - \delta_0)^2$, as $\delta \to \delta_0$.

According Redington immunization the assets flow has to be constructed so as the following equations fulfil: $V(A-L; \delta_0) = 0$; $V'(A-L; \delta_0) = 0$.

From these equations follows equation $V'(A; \delta_0) : V(A; \delta_0) = V'(L; \delta_0) : V(L; \delta_0)$ or $duration(A; \delta_0) = duration(L; \delta_0)$.

Comment. In order to choose simplest way we assume that interest is compounded continuously with force of growth δ_0. In such a case $duration(A; \delta_0) = V'(A; \delta_0) : V(A; \delta_0)$; $duration(L; \delta_0) = V'(L; \delta_0) : V(L; \delta_0)$.

Therefore the Redington immunization conditions have a clear financial interpretation. Namely, the asset flow is constructed so as:

[15] By definition *volatility* $V(A; \delta) := [ln\ V(A; \delta)]'_\delta$, *volatility* $V(L; \delta) := [ln\ V(L; \delta)]'_\delta$.

[16] Redington, F. M. Review of the Principles pf Life Office Valuations. Journal of the Institute of Actuaries, 1952. Vol. 78, pp. 286-315.

- the first, initial balance condition fulfils − at the interest rate δ_0 the present value of the assets flow equals the present value of the liabilities flow;
- the second, at the initial interest rate δ_0 the discounted mean term the assets flow equals the discounted mean term the liabilities flow.

If conditions fulfil then $V(A-L;\,\delta) = 0{,}5 \cdot V''(A-L;\,\delta_0) \cdot (\delta - \delta_0)^2 + o[(\delta - \delta_0)^2]$.

If we can, additionaly, obtain the holding of inequality $V''(A-L;\,\delta_0) > 0$, then portfolio A−L is immunized in the vicinity of interest rate δ_0 because of $V(A-L;\,\delta) > 0$ for any $\delta \neq \delta_0$, wich by module little differs from δ_0. By that the gain of capital depends of how big we are able to obtain the value of the second derivative $V''(A-L;\,\delta_0)$.

Next follows the criticism of Redington immunization. It was surprising how high attention to the Redington immunization dedicate, for example, authors of extra modern book "Leslie Jane Federer Vaaler, James W. Daniel. Mathematical Interest Theory. Second edition. − The Mathematical Association of America. Pearsons Prentice Hall, 2007". In the same time author of present paper has introduced the integral imunization and has proved that integral immunization is more efficient then Redington immunization and Redington immunization can be considered as special case of integral immunization. In particular, author has proved, that "famous" condition *duration*(A; δ_0) = *duration*(L; δ_0) is nonrelevant for cash flow A−L immunization. The integral immunization is examined in details and compared with Redington immunization in the book of author [16].

In serious textbooks like, for instance, in book "William F. Sharpe, Gordon J. Alexander, Jeffery V. Bailey. Investments. Fifth Edition. − Prentice Hall International, Inc., 1995" [10] and pthers [11], [12], [13] the statements proved by author of present paper are not formulated.

4. INTEGRAL IMMUNIZATION AND ORDINATE IMMUNIZATION

I offer two immunization methods: integral immunization and ordinate immunization.

But first we need to find the answer to the question: how to compare the efficiency of different immunization methods?

Suppose the market capitalization rate is δ_0. I offer to measure the level of immunization with areas under the graph of function $V(A-L;\,\delta)$ above the segment $[\zeta;\,\delta_0]$ and segment $[\delta_0;\,\eta]$, where $0 \le \zeta \le \delta_0 \le \eta$.

Definition. Suppose that cash flow A−L is immunized in the some vicinity of interest rate δ_0.

Let us the vector function $(LSQ(\zeta), RSQ(\eta)) := \left(\int_{\varsigma}^{\delta_0} V(A-L;\,\delta)\,d\delta,\ \int_{\delta_0}^{\eta} V(A-L;\,\delta)\,d\delta\right)$,

where $0 \le \zeta \le \delta_0 \le \eta$, call as bicriterion of immunization. Here $LSQ(\zeta)$ is left-side square and $RSQ(\eta)$ is right-side square. Immunization expects that these both squares are obtained so big as possible. The efficiency of immunization characterizes Pareto frontier − the set of Pareto efficient points ($LSQ(\zeta)$, $RSQ(\eta)$).

In order to perform practical calculations is handy to use the following notations.

Liabilities vector $L = (l_1\ l_2\ \dots\ l_n)$.

$(m\times n)$-matrix $B = (b_{ij})$, which rows are the cash flows $B_1, B_2, \dots, B_m$ associated with available fixed income securities (bills, notes, bonds).

Discount vector $D_0 := (\nu_0\ \nu_0^2\ \dots\ \nu_0^n)^T$, $\nu_0 := exp(-\delta_0)$.

Discount vectors $D_1 := (1^1\nu_0^1\ 2^1\nu_0^2\ \dots\ n^1\nu_0^n)^T$, $D_2 := (1^2\nu_0^1\ 2^2\nu_0^2\ \dots\ n^2\nu_0^n)^T$,

what we will call as the discount vectors for first derivative and for the second derivative correspondingly.

Let us take in account that $V(B_i;\,\delta_0) = B_i D_0$, $V'(B_i;\,\delta_0) = B_i D_1$, $V''(B_i;\,\delta_0) = B_i D_2$;
$i \in \{1, 2, \dots, m\}$.

Vector $X := (x_1\ x_2\ \dots\ x_m)$ what we will cal as content of the assets portfolio.

In these notations: $A = XB$, $XBD_0 = V(A, \delta_0)$, $XBD_1 = V'(A, \delta_0)$, $XBD_2 = V''(A, \delta_0)$,
$LD_0 = V(L, \delta_0)$, $LD_1 = V'(L, \delta_0)$, $LD_2 = V''(L, \delta_0)$.

Now we are going to obtain the expression of the immunization efficiency bicriterion

$(LSQ(\zeta), RSQ(\eta)) := \left(\int_{\varsigma}^{\delta_0} V(A-L;\,\delta)\,d\delta,\ \int_{\delta_0}^{\eta} V(A-L;\,\delta)\,d\delta\right)$, kur $0 \le \zeta \le \delta_0 \le \eta$.

At first we express V(A–L; δ) as function of interest rate δ and content of portfolio X:

$$V(A-L;\delta) = \sum_{j=1}^{n}\left(\sum_{k=1}^{m} x_k\, b_{kj} - l_j\right) e^{-\delta j} = (XB - L)D =: f(\delta, X).$$

Let us stress that discount vector $D := (\nu\ \ \nu^2\ \ ...\ \ \nu^n)^T$ depends from rate of capitalization δ; $\nu := \mathit{exp}(-\delta)$.

It is easy to see that for any $j \in \{1, 2, ... , n\}$ the following equations hold:

$$\int_{\varsigma}^{\delta_0} e^{-\delta j}\, d\delta = -\frac{1}{j}\left(e^{-\delta_0 j} - e^{-\varsigma j}\right) = \frac{1}{j} e^{-\delta_0 j}\left(e^{(\delta_0-\varsigma)\cdot j} - 1\right),$$

$$\int_{\delta_0}^{\eta} e^{-\delta j}\, d\delta = -\frac{1}{j}\left(e^{-\eta j} - e^{-\delta_0 j}\right) = \frac{1}{j} e^{-\delta_0 j}\left(1 - e^{(\delta_0-\eta)\cdot j}\right).$$

As result we get the expressions of left-side square LSQ(ζ) and right-side square RSQ(η):

$$LSQ(\zeta) := \int_{\varsigma}^{\delta_0} f(\delta, X)\, d\delta = (XB - L)D_{LSQ},\quad RSQ(\eta) := \int_{\delta_0}^{\eta} f(\delta, X)\, d\delta = (XB - L)D_{RSQ},$$

where D_{LSQ}, D_{RSQ} are the following vectors-columns:

$$D_{LSQ}(\zeta) := \left(\frac{1}{1}\nu_0^1\left(e^{(\delta_0-\varsigma)\cdot 1} - 1\right)\ \ \frac{1}{2}\nu_0^2\left(e^{(\delta_0-\varsigma)\cdot 2} - 1\right)\ ...\ \frac{1}{n}\nu_0^n\left(e^{(\delta_0-\varsigma)\cdot n} - 1\right)\right)^T$$

$$D_{RSQ}(\eta) := \left(\frac{1}{1}\nu_0^1\left(1 - e^{(\delta_0-\eta)\cdot 1}\right)\ \ \frac{1}{2}\nu_0^2\left(1 - e^{(\delta_0-\eta)\cdot 2}\right)\ ...\ \frac{1}{n}\nu_0^n\left(1 - e^{(\delta_0-\eta)\cdot n}\right)\right)^T$$

$\nu_0 = e^{-\delta_0}$.

Let us introduce the concept of "ordinate immunization". Empirical testing shows that is possible to obtain efficient immunization during maximization especially chosen ordinates of function V(A−L; δ). Such approach could be useful, for instance, if investor predicts one percent point increasing of the interest rates as more believable then one percent point decreasing of it. In order to determine the goal-directed content of portfolio investor calculates Pareto frontier of the set $\{(V(A-L;\ \delta_0-0{,}01),\ V(A-L;\ \delta_0+0{,}01))\ |\ XBD_0 = LD_0,\ X \geq O\}$ as a decision making tool.

Do to limitations of paper volume the concept of ordinate immunization is here not discussed.

As it is mentioned before the integral immunization, introduced by author, is compared with Redington immunization in the book of author [16]. The following conclusions are obtained.

1. The Redington immunization what is based on the Taylors' formula and concept of duration can be considered as special case of more general immunization method − integral immunization. Namely, the Redington immunization result can be find as separate point on the Pareto frontier which is a tool of integral immunization.

2. Integral immunization method is flexible in case if investor is interested in so called immunization according expectations. The bicriterion can be vary, for instance, in forms RSQ = 2 LSQ or LSQ = 2 RSQ according the prognosis of the interest rate possible changes.

3. The portfolio construction, management and protection conception based on the idea of immunization leads to the following consequence: the financial market is for portfolio owner the more profitable during some period of time the more interest rate volatility is. It implies that portfolio must be re-immunized on a regular basis.

5. IMMUNIZED QUASI-ARBITRAGE AS RESULT OF GOAL-DIRECTED INVESTMENT AND FINANCING PROJECTS DIVERSIFICATION

a. Investment and financing diversification in order to obtain quasi-arbitrage

Let us examine the simultaneous investment and financing programming in case of determined cash flows. In the columns of Table 1 available investment and financing projects in form of cash flows are given. Or goal is to construct the program of agents' actions what leads to the immunized quasi-arbitrage. This agent is quasi-arbiter, because of being capable to make more goal-directed portfolio construction with help of borrowed and lent money to compare with another investors.

Table 1

The available investment and financing project in form of cash flows

time	INV_1	...	INV_n	FIN_1	...	FIN_m
0	$a_{0\,1}$	...	$a_{0\,n}$	$b_{0\,1}$	...	$b_{0\,m}$
1	$a_{1\,1}$	...	$a_{1\,n}$	$b_{1\,1}$	...	$b_{1\,m}$
...	...	...	...	...	...	...
τ	$a_{\tau\,1}$	...	$a_{\tau\,n}$	$b_{\tau\,1}$	...	$b_{\tau\,m}$

All projects extend τ periods. Let us call the matrix INV := ($a_{i\,j}$) as investment matrix, but matrix FIN := ($b_{i\,j}$) − as financing matrix.

In the Lutz Krushwitz model[17] the exogenous cash flow M := $(m_0\ \ m_1\ \ ...\ \ m_\tau)^T \geq O$ what belongs to agent is included. In the simplest case $m_0 > 0$, $m_1 = 0$, $m_2 = 0$, ... , $m_\tau = 0$.

It means that at the initial moment agent invests his own money m_0, but after that some other investments of private money is not proposed.

Let us introduce the following notations.

Let us call the vector $X = (x_1\ \ x_2\ ...\ \ x_n)^T$ as investment plan, the vector $Y = (y_1\ \ y_2\ ...\ \ y_m)^T$ as financing plan, but the pair of vectors (X, Y) as investment and financing strategy.

In order the plans X, Y would be feasible they have to satisfy the technological conditions

O ≤ X ≤ X^, O ≤ Y ≤ Y^.

We assume that investment and financing plans are arbitrary divisible. It means that x_1, x_2, ... , x_n, y_1 y_2 ... y_m are real numbers.

As result of strategy (X, Y) and exogenous investment M we obtain the endogenous cash flow

INV X + FIN Y + M. The cash flow INV X + FIN Y can be interpreted as intertemporal cash bundle what agent can buy paid for it the intertemporal cash bundle M.

Therefore the set of all intertemporal cash bundle available for agent is

Z(M) := { Z | Z = INV X + FIN Y + M, O ≤ X ≤ X^, O ≤ Y ≤ Y^ }.

In order to investigate the arbitrage or at least quasi-arbitrage possibilities we will consider the case M = O, namely, investor does not invest his own money at all. Investments will be done solely used borrowed money. However we prefer term "quasi-arbitrage" because of transactions costs, of course, present in this investing and financing process.

In the literature we can meet the wide discussions about criterions of quality of the endogenous cash flow $Z = (z_0\ \ z_1\ ...\ \ z_\tau)^T$. In this paper according to the concept of immunization we will try to reach maximum of net present value $V(Z, \delta_0)$, where δ_0 is initial interest rate. After that we will immunized $V(Z, \delta_0)$ with help of integral method. Here left-side square $LSQ(\zeta)$ and right-side square $RSQ(\eta)$ can be expressed as follows:

$LSQ(\zeta) = (INV\ X + FIN\ Y)\ D_{LSQ}$, $RSQ(\eta) = (INV\ X + FIN\ Y)\ D_{RSQ}$.

b. Example of immunized quasi-arbitrage

We are going to illustrate the problem with help of simultaneous investment and financing programming fictitious case borrowed of Dr. Lutz Krushwitz pithy book " Lutz Kruschwitz. Investitionsrechnung. 8., Neu bearbeitete Auflage. − R. Oldenbourg Wissenschaftsverlag, 2000" [14]. Of course, it is easy to construct another fictitious case, but author prefers to use exactly this because of respect and gratitude to the Dr. Lutz Krushwitz, which books helps us join the financial theory. Author had already utilized Dr. Lutz Krushwitz fictitious case as tool for empirical testing and illustrating the different theoretical conclusions in the papers [17], [18], [19].

[17] Lutz Kruschwitz. Investitionsrechnung. 8., Neu bearbeitete Auflage. − R. Oldenbourg Wissenschaftsverlag, 2000. Лутц Крушвиц. Инвестиционные расчеты. – Питер, 2001.

Let us assume that at the initial moment the market capitalization rate is δ_0 = 0,12.
In the Table 2 the available investment projects in form of associated cash flows are exposed.
In the Table 3 the internal rates of return IRR of the investment cash flows are exposed.

Table 2

Investment matrix INV

time	INV1	INV2	INV3	INV4	INV5	INV6	INV7	INV8
t = 0	0	–800	–700	–300	–100	0	0	0
t = 1	–500	80	500	700	106	–100	0	0
t = 2	–900	160	300	350	0	106	–100	0
t = 3	1250	320	–200	170	0	0	106	–100
t = 4	350	520	220	–1090	0	0	0	106

Table 3

Internal rates of return of the investment cash flows

	INV_1	INV_2	INV_3	INV_4	INV_5	INV_6	INV_7	INV_8
IRR =	0,0883	0,1002	0,1025	0,0931	0,0600	0,0600	0,0600	0,0600

In the table 4 the available financing projects in form of associated cash flows are exposed.

Table 4

Financing matrix FIN

time	FIN_1	FIN_2	FIN_3	FIN_4	FIN_5	FIN_6
t = 0	1000	600	100	0	0	0
t = 1	–80	0	–110	100	0	0
t = 2	–388	0	0	–110	100	0
t = 3	–388	0	0	0	–110	100
t = 4	–388	–832	0	0	0	–110

In the table 5 the internal rates of return IRR of the financing cash flows are exposed.

Table 5

Internal rates of return of the financing cash flows

	FIN_1	FIN_2	FIN_3	FIN_4	FIN_5	FIN_6
IRR =	0,0800	0,0852	0,1000	0,1000	0,1000	0,1000

In the case examined the investment plan is vector $X = (x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ x_7\ x_8)^T$, financing plan is vector $Y = (y_1\ y_2\ y_3\ y_4\ y_5\ y_6)^T$.

In our example the conditions $x_1 \leq 1$, $x_2 \leq 1$, $x_3 \leq 1$, $x_4 \leq 2$; $y_2 \leq 1$ must hold.
As result of actions according strategy (X, Y) the agent obtains the endogenous cash flow Z = INV X + FIN Y.
Therefore the following linear programming problem arises:
$max\{V(Z; \delta_0 = 0{,}12) \mid Z = INV\ X + FIN\ Y;\ X \geq O,\ Y \geq O;\ x_1 \leq 1,\ x_2 \leq 1,\ x_3 \leq 1,\ x_4 \leq 2;\ y_2 \leq 1\}$.
Let us denote the solution as Z*. Empirical calculations show that quasi-arbitrage is possible.
After that with help of integral immunization we are going to correct the previous solution − strategy (X, Y) and obtain immunized endogenous cash flow $Z^{\#}$.
In order to do this let us take $\zeta = 0{,}11$; $\eta = 0{,}13$.
We resign from maximal value $V(Z^*, \delta_0)$ and solve the problem
$max\{ LSQ(\zeta) \mid V(Z, \delta_0) \geq V(Z^*, \delta_0) - \varepsilon;\ Z = INV\ X + FIN\ Y;\ LSQ(\zeta) = RSQ(\eta);\ X \geq O,\ Y \geq O;\ x_1 \leq 1,\ x_2 \leq 1,\ x_3 \leq 1,\ x_4 \leq 2;\ y_2 \leq 1 \}$.

The immunized quasi-arbitrage strategy is exposed in the second column of the table 6, corresponding endogenous cash flow is exposed in the lower row of the Table 6.

Table 6

The strategy of the imunized arbitrage

	Time periods	t = 0	t = 1	t = 2	t = 3	t = 4
x_1 =	0,26	0	-500	-900	1250	350
x_2 =	1,00	-800	80	160	320	520
x_3 =	1,00	-700	500	300	-200	220
x_4 =	0,00	-300	700	350	170	-1090
x_5 =	0,00	-100	106	0	0	0
x_6 =	0,00	0	-100	106	0	0
x_7 =	0,00	0	0	-100	106	0
x_8 =	0,98	0	0	0	-100	106
y_1 =	1,50	1000	-80	-388	-388	-388
y_2 =	0,00	600	0	0	0	-832
y_3 =	0,00	100	-110	0	0	0
y_4 =	1,32	0	100	-110	0	0
y_5 =	0,00	0	0	100	-110	0
y_6 =	0,00	0	0	0	100	-110
Endogenous cash flow	$Z^{\#}$ =	0,00	463,19	-500,00	-237,57	352,71

In the Table 7 the net present values of the endogenous cash flow for different interest rates are exposed. In the Figure 1 the graph of function $V(Z; \delta)$ is depicted.

Table 7

The net present values of the endogenous cash flow Z

δ	0,05	0,06	0,07	0,08	0,09	0,10	0,11	0,12	0,13	0,14	0,15	0,16	0,17	0,18
V(Z; δ)	72,47	71,77	71,20	70,74	70,41	70,18	70,04	70,00	70,04	70,16	70,35	70,60	70,92	71,29

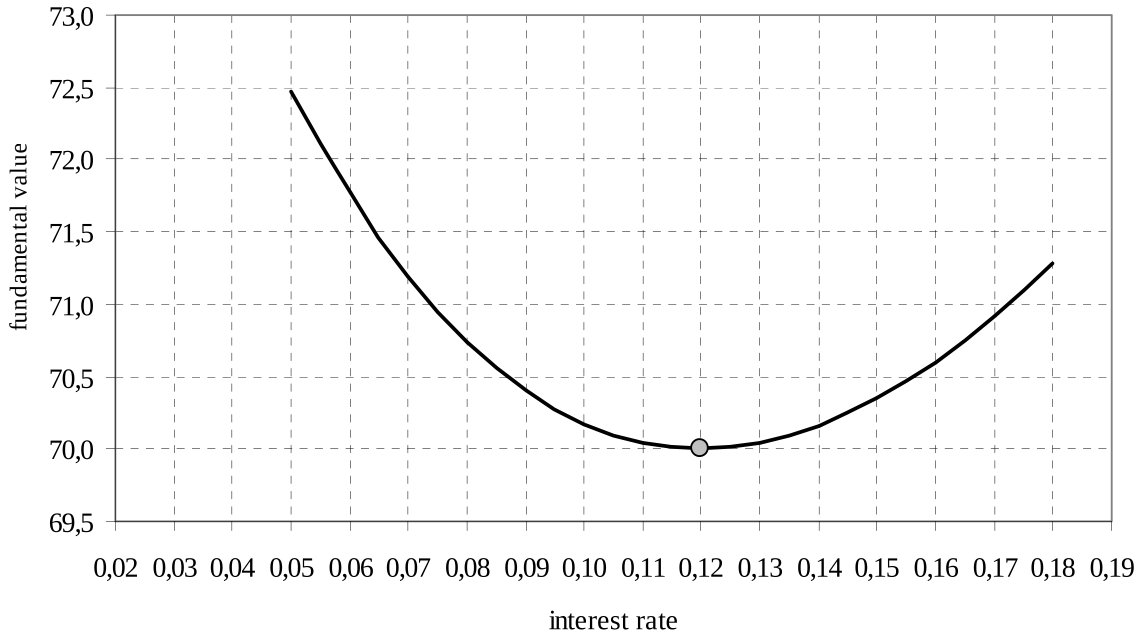

Figure 1. The graph of the function $V(Z; \delta) = 463{,}19\ v - 500\ v^2 - 237{,}57\ v^3 + 352{,}71\ v^4$; $v = exp(-0{,}12)$.

Thus we have empirically showed that in the simultaneous investment and financing programming fictitious case borrowed from Dr. Lutz Krushwitz is possible to construct immunized quasi-arbitrage strategy what result is the endogenous cash flow Z = (0,00 463,19 −500,00 −237,67 352,71). The fundamental value of Z by discounting with interest rate δ_0 = 0,12 is 70,00. If interest rate δ fluctuates around the initial value δ_0 = 0,12 the fundamental value of Z increases.

Table 8 exposed the relationship between immunized quasi-arbitrage investment plan and internal rates of return of available investment projects. Table 9 exposed the relationship between immunized quasi-arbitrage financing plan and internal rates of return of available financing projects. It is important to observe, that for quasi-arbiter the internal rate of return is not the only criterion for including the investment project or financing project in portfolio. For instance, IRR(INV_4) = 0,0931 > IRR(INV_1) = 0,0883, in spite of that project INV_1 is included in investment plan, but INV_4 is not included in plan; IRR(FIN_4) = 0,1000 > IRR(FIN_2) = 0,0852, but project FIN_4 is included in financing plan and FIN_2 is not included in plan.

Table 8

Relationship between investment plan and internal rates of return

	INV_1	INV_2	INV_3	INV_4	INV_5	INV_6	INV_7	INV_8
Investment plan	0,26	1	1	0	0	0	0	0,98
IRR	0,0883	0,1002	0,1025	0,0931	0,0600	0,0600	0,0600	0,0600

Table 9

Relationship between financing plan and internal rates of return

	FIN_1	FIN_2	FIN_3	FIN_4	FIN_5	FIN_6
Financing plan	1,50	0	0	1,32	0	0
IRR	0,0800	0,0852	0,1000	0,1000	0,1000	0,1000

6. CONCLUSION

The most important topics in modern economic science are associated with asymmetric information presence in the market and different dramatic consequences which follow from it. The roots of the concept of quasi-arbitrage, offered by author in present paper, also lead to asymmetric information. The main goal of this paper is to find an answer to the question: is it possible to construct simultaneous financing and investment program which is both quasi-arbitraged and immunized. The problem is solved by utilizing integral immunization tool, created by author.

In the opinion of author, the original concept of quasi-arbitrage and original tool of integral immunization presented in this paper must take significant place in microeconomics and investment theory and practice.

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